Designing and testing a high-bandwidth 2-D wavefront sensor for Aero-optics

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ABSTRACT

A novel two-dimensional high-bandwidth Shack-Hartmann wavefront sensor was designed, addressing the high temporal bandwidth of optical aberrations caused by compressible flows. The principle of operation and modifications from an earlier version of the wavefront sensor are presented and compared to a commercially available wavefront sensor. A wavefront reconstruction algorithm is derived. The high-temporal resolution and spatial resolution of the sensor is demonstrated. A two-dimensional, acoustically forced heated jet was used to test the sensor.

Keywords: Aero-optics, Shack-Hartmann, Wavefront sensor, Adaptive optics, Position sensing device (PSD)

INTRODUCTION

When a perfectly collimated, planar beam of light propagates through a field with variable index-of-refraction it becomes distorted, resulting in significant reduction in the optical intensity in the far field, and degradation in overall light transmitting system performance. When the transmitting system is airborne, and the variable index-of-refraction flow is close to the aircraft (∼1-2 aperture lengths), the effect is called the aero-optical problem1.

Contrary to atmospheric optical aberrations, which are caused by temperature gradients in the far field and characterized with low temporal frequencies of less than a kilohertz, the aero-optically active aberration area is in the near-field and caused by density gradients due to boundary layers, separated shear layers, and wakes around an aircraft, see Fig. 1; these aberrations are characterized with high temporal frequencies, typically around several kilohertz. In this context, the terms 'near-field' and 'far-field' do not refer to the Fresnel and Fraunhofer diffractions as defined by Fresnel number, but, to distortion regions with respect to the beam aperture size, A; aero-optical effects are caused by a turbulent flow around the airborne platform and characterized by a region on the order of several apertures, while atmospheric propagation effects are due to temperature fluctuations in the atmosphere, extending from just beyond the aircraft to the target. The index-of-refraction, n, fluctuations can be related to the density, ρ, fluctuations by Gladstone-Dale relation5:

\[ n = 1 + \rho K_{GD} \] (1)

where, \( K_{GD} \) is the Gladstone-Dale constant.

To measure optical aberrations, compensate for the intensity loses, and maximize propagated beam energy on a target, a closed loop adaptive optics system (AO) is typically used; such system senses the aberration, using a wavefront sensor (WFS), reconstructs the wavefront phase to estimate and predicate phase aberrations using a reconstructor, and constructs a conjugate correction using a deformable mirror (DM)6.

Fig. 1. Flow over a turret/fairing combination.
It was shown by Nightingale et al.\(^3\) that in order to develop an adaptive-optics system which is capable of operating in a real flight scenario, a wavefront sensor with a real time framing rate that is on the order of 100 times the bandwidth of an aberration is required. As it was mentioned, aero-optical disturbances are characterized with high temporal frequencies around several kilohertz, hence a wavefront sensor that operates with a real time sampling rate of hundreds of kilohertz is required. Current state-of-the-art conventional adaptive-optics systems, are limited in bandwidth to hundreds of hertz disturbances correction, making them unsuitable to compensate aero-optics-related intensity losses; different approaches need to be considered to compensate for the aero-optical aberration effects on the transmitted beam.

In addition, the wavefront sensor should provide wavefronts with a minimum delay between sensing and calculating wavefronts for adaptive-optics system to be stable\(^7\). High framing rate and fast wavefront reconstruction are above the current capability of most of the commercially available, digital charge-coupled devices (CCD) wavefront sensors; there is the need to design a high bandwidth wavefront sensor that does not use CCD technology.

With this in mind, a new 2-D high-bandwidth Shack-Hartmann wavefront sensor (SHWFS) was designed, capable of acquiring temporal data at rates greater than 100 kHz, in addition to a spatial resolution of one-tenth of the aperture size. This novel sensor can be used to address high temporal bandwidth aero-optical aberrated flows that cannot be addressed by the currently available wavefront sensors. This sensor is a third generation of a sensors originally developed by Cicchello, Hugo, Cheung, Jumper, Mminiti, Preston and Cavaliere under a joint STTR program between Notre Dame and Oceaniti\(^7\).

Wavefront aberrations are commonly quantified using optical path difference (OPD), which is the spatial-mean-removed optical path length (OPL). The OPL is defined as the path integral of the index-of-refraction field, \(n\), along the beam path, \(s\),

\[
OPL(t, x, y, z) = \int_{s_1}^{s_2} n(t, x, y, z)ds
\]

(2)

OPD is obtained by removing the spatial-mean over the aperture,

\[
OPD(t, x, y, z) = OPL(t, x, y, z) - \bar{OPL}(t, x, y, z)
\]

(3)

Spatial OPD root-mean-squares (OPD\(_{\text{rms}}\)) and OPD peak-to-peak (OPD\(_{\text{p-p}}\)) values are calculated to quantify the deviation from a perfectly collimated beam.

To verify the accuracy of the newly designed wavefront sensor, a benchmark experiment was conducted in the heated jet facility at Hessert Laboratory, University of Notre Dame. A low-speed commercially available 2-D wavefront sensor was used to make benchmark measurements. Streamwise realizations of roll-up structures were correctly captured with both the low and high-speed wavefront sensors.

Section 2 of the paper presents a detailed description of the Shack-Hartmann wavefront sensor’s method of operation. A comprehensive discussion of the modifications on the conventional sensor, which improve the sensor’s accuracy, sensitivity and sampling rate and comparison to the previous generation of designed high bandwidth wavefront sensor, is presented in Section 3. A wavefront phase reconstruction algorithm, which was developed for the new sensor layout configuration, is derived and presented in Section 4. A brief description of the heated jet facility is presented in Section 5. A description of the benchmark test setup and results of comparison between the newly designed wavefront sensor and the commercially available one are presented in Section 6. The paper concludes with a discussion of the presented results and future work.

2. SHACK-HARTMANN WAVEFRONT SENSORS

In the early 1900s Hartmann proposed a sensor to measure wavefront aberrations by measuring the transverse ray aberration in the image plane\(^5\). Based on Huygen’s principle (which states that a ray of light travels normal to its associated optical wavefront\(^6\)), Hartmann placed a mask with holes in front of the incoming beam, and a photographic plate a known distance, \(d\), from the mask. See in Fig. 2, left.

By measuring the displacement, \(e\), of each light ray from its unaberrated position, and knowing the distance from the mask to the photographic plate, \(d\), it is possible to find the local slope, \(\theta_x\), and reconstruct the wavefront phase by integration. The main disadvantage of Hartmann plate test is the requirement for a unique mask for each tested optics.
The Shack-Hartmann wavefront sensor is a modification of the classical Hartmann plate test. For this sensor, the mask was replaced by a 2-D lenslet array with a focal length, \( f \), fixed in front of a position sensing detector (PSD), such as CCD or photodiodes, instead of a photographic plate, as shown in Fig. 3. The lenslet array spatially divides the incoming beam into an array of sub-apertures, where the deflection angle, \( \theta \), over each sub-aperture represents the average wavefront local tilt, and independent of the intensity pattern across the sub-aperture. Each lenslet sub-aperture is associated with a PSD which evaluates the local transverse dot displacement, \( \epsilon \), directly related to the local wavefront gradient, \( \theta \), of the wavefront by the equation

\[
\theta \approx \frac{\epsilon}{f} = \nabla(OPL)
\]

(4)

The wavefront is reconstructed from the slopes at finite points using either direct or iterative integration methods to solve Eq. (4) for the entire 2-D array.\(^{15-17}\) The spatial resolution of the sensor determined by the lenslet focal length, \( f \), and sub-apertures spacing. The sensor sensitivity, \( \theta_{\text{min}} \), is determined by the minimum detectable change in the focal spot position, \( \epsilon_{\text{min}} \), based on the equation

\[
\theta_{\text{min}} = \frac{\epsilon_{\text{min}}}{f}
\]

(5)

hence, increasing the focal length, \( f \), leads to improving the measurement sensitivity. The sensor dynamic range, \( \theta_{\text{max}} \), is defined as the largest measurable slope and is defined by the sub-aperture size, \( d \), as

\[
\theta_{\text{max}} = \frac{d}{f}
\]

(6)

As Eqs. (5) and (6) indicate, there is a trade-off between the measurement sensitivity and the dynamic range, where a large lenslet focal length may improve the measurement sensitivity but reduce the dynamic range, and vice versa. The temporal resolution is limited by the sampling rate of the detectors used to determine the spots displacements.

Fig. 3. Shack-Hartmann wavefront sensor: principle of operation.

3. HIGH-BANDWIDTH WAVEFRONT SENSOR

A number of wavefront sensors were developed by the aero-optics group in Notre Dame, the first was a 1-D SABT (Small-Aperture Beam Technique) wavefront sensor introduced by Jumper et al.\(^{16}\) which demonstrated a wavefront
capturing rate up to 100 kHz. A second generation of 8x8 sub-apertures high-bandwidth Shack-Hartmann wavefront sensor (SHWFS) was designed under a joint STTR between Notre Dame and Occam. Despite the high capturing rate, this wavefront sensor was associated with number of disadvantages, mainly:

- Tetra-lateral position sensing detectors were used to detect the dot displacement and these detectors exhibit a non-linear response between the spot position and output voltage near the detector’s edges.
- The 8x8 sub-aperture layout was obtained by manually combining four 4x4 sub-apertures substrates, introducing alignment problem.
- A square 80x80 mm custom-made lenslet array with a focal length of 0.5 m was used, increasing the price of the system and the optical bench layout size.
- Four identical voltage summation and subtraction boxes were used, increasing the electronic rack size, and BNC cables were used to connect the system components, inducing electronic noise and further adding to the complexity of the system.

The design of the new sensor was driven by addressing these disadvantages and presented in the following sub-sections.

3.1. Position Sensing Devices (PSD)

As it was mentioned above, aero-optics effects are associated with high temporal frequencies exceeding the current framing capability of commercially available wavefront sensors based on CCD detectors; these sensors grab the spots image and then usually use a time-consuming, Gaussian fitting, data-reduction algorithm to approximate the center portion of the spot image to find the spots centroid. It should be mentioned that high-framing rate CCD cameras do exist; however, running these cameras at high sampling frequencies reduces the pixel count available per sub-aperture. Theoretically at least four pixels are needed to resolve the focus spot centroid as a quad cell. Shack-Hartmann sensors using these high-bandwidth CCD arrays have been constructed for research demonstrations; however, there are still complications in turning these sensors into real-time sensors.

The newly designed high bandwidth wavefront sensor has 68 duo-lateral analog position sensing devices from Pacific Silicon Sensor Inc. with an active area of 4x4 mm and a maximum rise time of 1 µs. While the tetra-lateral PSD’s exhibits a position non-linearity near the sensor edges, resulting in a larger detection error, the duo-lateral PSDs have a linear response over the entire sensor area with very small position detection error. A detailed description of the duo-lateral PSD principle of operation and characteristics can be found in Ref. 13. Here, we provide a brief description only.

The duo-lateral position sensing device has four electrodes placed at its edges, with a p-n junction between two resistive layers, where the photocurrent, generated by the incident light spot, is divided into two parts in each layer, providing a continuous position data. The generated currents are transformed into output voltages by current-to-voltage transimpedance amplifiers, \( V_1 \) and \( V_2 \) for \( x \)-direction, and \( V_3 \) and \( V_4 \) for the \( y \)-direction, the relation between the voltages gives the light spots centroid through the formulas

\[
\text{Div } X = K_x \cdot \frac{V_1 - V_2}{V_1 + V_2} = K_x \cdot \frac{\text{Dif } X}{\text{Sum } X} \quad \text{and} \quad \text{Div } Y = K_y \cdot \frac{V_3 - V_4}{V_3 + V_4} = K_y \cdot \frac{\text{Dif } Y}{\text{Sum } Y}
\]  

(7, 8)

where, \( K_x \) and \( K_y \) are calibration coefficients which are constant over the area of the detector. For each detector the voltage summation and subtraction are performed using electronic chips, defining an additional two parameters for each direction, \( \text{Sum } X \) and \( \text{Dif } X \) for the \( x \)-direction and similarly \( \text{Sum } Y \) and \( \text{Dif } Y \) for the \( y \)-direction. The signals divisions for the sensor are performed by post-processing software, but an all-analog summation-subtraction-division electronic board is currently being designed to provide real-time spot locations.

Prior to each data run, the detectors were calibrated by shifting the entire detector board through a series of \( x \)-and \( y \)-positions and the \( x \)-and \( y \)-voltages were measured as a function of the focal spot location on each PSD sensor. To do this, the detector board was mounted on a two-dimensional \( X-Y \)-linear translation stage and moved over a pre-defined two-dimensional displacement grid with respect to the fixed light beam; each sensor can be represented by a 2x2 matrix, relating the two division voltages of each detector, \( \text{Div } X \) and \( \text{Div } Y \) defined in Eqs. (7)-(8), to the two-dimensional \((x, y)\) spot position.
\[
\begin{bmatrix}
X \\
Y
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\begin{bmatrix}
\text{Div}X \\
\text{Div}Y
\end{bmatrix}
\] (9)

The matrices were saved and used to convert the four acquired voltage signals from each detector into a spot displacement, \(e\). Due to the photodiodes characteristics, the spots displacement-voltage relation is linear; so Eq. (9) has linear terms only, which simplifies the calibration procedure.

Replacing the charge-coupled device (CCD) array with an array of analog PSDs offered an accurate continuous spot position measurement, and a fast response time that could not be achieved with the commercially available wavefront sensors. The time-consuming CCD data-reduction algorithm was replaced by a simple spot analog centroid measurement, achieving sampling frame rates greater than 100 kHz, and limited only by the sampling speed of the A/D conversion boards.

3.2. Detectors Layout

The 68 detectors were mounted on a single substrate and arranged in a 85 mm diameter circular pattern (10x10 with removed corners) with 9 mm distance between each two adjacent PSDs, see Fig. 4, left. The new single-board detector arrangement eliminated alignment problems which existed in the previous 8x8 wavefront sensor\(^{11,12}\) due to the need of combining four 4x4 units to obtain the overall 8x8 detectors square layout, see Fig. 4, right. In addition, the previous 8x8 wavefront sensor required a custom-made square lenslet array to match the lenses with the detectors resulting in a difficult alignment and, for round beams, not utilizing the corner sensors. The circular detector pattern of the new wavefront sensor is more relevant to optical tests were a circular beam shape is usually used, and if needed, it can be directly matched with a round deformable mirror.

![Fig. 4. Layout of the 10x10 wavefront sensor, left. Layout of the previous 8x8 wavefront sensor, right.](image)

3.3. Electronics

Four analog voltage signals per sensor or a total number of 272 analog voltage signals, are sampled by three data acquisition boards from Microstar Laboratories, Inc., controlled by a data acquisition computer, also used to post-process the acquired data. The sensor electronic design was improved by designing a single multi-layer board with a smaller summation/subtraction chips, and directly attached to the detector board, while the transimpedance amplifiers were mounted on the same board, decreasing the number of required connection wires, reducing electronic noise, and electronic rack size significantly, compared to the previous 8x8 wavefront sensor\(^{11,12}\) where BNC cables were used to connect the system components and four summation/subtraction breakout boxes were used. The new summation/subtraction board is shown in Fig. 5, left, a photo of the old electronic rack data acquisition system is shown in Fig. 5, right.
3.4. Lenslet, Sensor Sensitivity and Dynamic range.

A key requirement of a wavefront sensor system is to have a small optical bench size and a maximized sensitivity. A monolithic lenslet array with 1 mm pitch between lenslets, 55 mm focal length and 25 mm diameter size, from Adaptive Optics Associates, Inc. was used, reducing the lenslet array size significantly, compared to the previous generation of 8x8 wavefront sensor\textsuperscript{11,12}, where a custom made 80x80 mm lenslet array with a focal length of 0.5 m was used. To improve the sensor sensitivity, and re-image the dot pattern onto the detector board, a re-imaging telescope was designed; the telescope uses three off-the-shelf lenses, two negative ones and a positive one, to re-image the lenslet array original image plane onto the detector plane with 9X magnification with minimal aberrations. With the telescope the lenslet focal length is increased from \( f \) to an effective focal length of \( M \cdot f \). Figure 6, right, gives a schematic of the telescope optical layout. With the re-imaging telescope Eq. (5), describing the sensor sensitivity, can be re-written as,

\[
\theta_{\text{min}} = \frac{\varepsilon_{\text{min}}}{M \cdot f}
\]

where, \( M \) is the telescope magnification, \( f \) is the lenslet focal length, and \( \varepsilon_{\text{min}} \) is the minimum detectable focal spot displacement in the image plane of the wavefront sensor due to the local aberrations in the wavefront. Based on Eq. (10), aberrations that otherwise cannot be detected without the re-imaging telescope due to the PSD physical limitation, now, could be magnified and detected. Fig. 6, left presents the designed telescope case, containing the lenslet array, and the three lenses. Based on the detectors specification, the noise-limited spot-position resolution at 632 nm, 0.5 \( \mu \)W is 0.06 \( \mu \)m, which corresponds to 0.12 \( \mu \)rad noise-limited slope for the 55 mm focal length lenslet array. Although, it should be mentioned, that the noise-limited resolution value was given by the manufacturer and based on testing done under very controlled conditions, resulting in a very small noise-level. In reality, the spot is typically limited to the minimum voltage resolution of an A/D converter; using Eqs. (7)-(8), the accuracy in measuring the spot positions for the current system was estimated as \( \sim 1.5 \mu \)m \( \mu \)r.

The sensor was tested with a 2W continuous Nd:YAG laser, operating at \( \lambda = 532 \text{ nm} \) wavelength. Based on the characteristic photo sensitivity versus wavelength plot, shown in Fig. 7, using a HeNe laser, operating at \( \lambda = 632 \text{ nm} \)
wavelength, would improve the sensitivity by approximately 50%, and operating in the near infra-red (IR) region, using a laser with $\lambda = 1064\text{ nm}$ wavelength as an example, would improve the sensitivity by 120%.

![Graph showing characteristic photosensitivity versus wavelength plot](image)

**Fig. 7.** Characteristic photosensitivity versus wavelength plot

Based on Eq. (5) a comparison can be performed between the sensitivities of the two sensors. For the same input beam with a diameter of $D = 1^\circ$, and recalling that the previous wavefront sensor had a lenslet array size of $D_{10x10} = 80\times80\text{ mm}$ and focal length of $f_{10x10} = 0.5\text{ m}$, while the new wavefront sensor lenslet array active area is $D_{10x10} = 10\text{ mm}$ diameter and focal length of $f_{10x10} = 55\text{ mm}$ with $M = 9\times$ telescope, we calculate

$$\frac{\theta_{\text{min}}^{\text{new}}}{\theta_{\text{min}}^{10x10}} = \frac{D_{10x10}}{D_{\text{new}}} \cdot \frac{D_{\text{min}}}{D_{10x10}} \sim 0.8$$

(11)

The main reason for sensitivity improvement is due to the fact that for the new wavefront sensor the $1^\circ$-beam is reimaged to 10 mm, resulting in increasing local slopes based on Lagrange invariant$^2$, while for the previous wavefront sensor the beam had to be expanded to fit the 80x80 mm square lenslet array, thus decreasing the slopes and reducing the resolution.

The trade-off between the sensor sensitivity and dynamic range was addressed by using different lenslet arrays. Currently the sensor has two additional lenslet arrays with focal lengths of 17 mm and 45 mm. As mentioned before, the dynamic range is defined by the detector physical size, defining the maximum dynamic range, $\theta_{\text{max}}$, as

$$\theta_{\text{max}} = \frac{d}{2 \cdot M \cdot f}$$

(12)

where, $d$ is the detector side length size (the actual active area could be limited to the central 80% of the 4x4 mm detector physical area size). For $d = 4\text{ mm}$, $f = 55\text{ mm}$ and $M = 9$ a $\theta_{\text{max}} \sim 4\text{ mrad}$ was calculated, while using a lenslet with focal length of $f = 17\text{ mm}$ could improve $\theta_{\text{max}}$ to $\sim 13\text{ mrad}$. The theoretical ratio between the largest measurable slope, $\theta_{\text{max}}$, to the smallest detectable change in wavefront slope, $\theta_{\text{min}}$, for the 55 mm focal length lenslet array is $\sim 3 \cdot 10^4$, in reality, the ratio is smaller, but still large enough for most applications.

![Spot intensity pattern contour and intensity profile in x- and y- direction](image)

**Fig. 8.** Typical spot intensity pattern contour and intensity profile in x- and y- direction

Proc. of SPIE Vol. 7466 746602-7
In our case, where the lenslets have a square shape sub-aperture, the diffraction-limited spot size (although the shape of the central order of a sinc² pattern is not radially symmetric, the distance between the zeros along one of the axes is used here to approximate the spot size) is

\[ \sigma = \frac{2 \cdot \lambda \cdot M \cdot f}{p} \]  \hspace{1cm} (13)

where \( p \) is the pitch between sub-apertures. For the 55 mm focal length lenslet array, and \( p = 1 \text{ mm} \), \( \sigma \) is equal to \( \sim 0.5 \text{ mm} \). An intensity pattern contour of a single spot, and intensity profile in \( x \)- and \( y \)- direction are shown in Fig. 8.

Conventionally, another dynamic range limitation could arise due to spot overlapping causing a cross-talk between two adjacent Area of Interest (AOI), as in the case with CCD detectors. In our case, this scenario is not relevant due to the large pitch between each two adjacent PSDs (9 mm), still, the light spot can leave the detector active area due to larger wavefront slope, defined by \( \theta_{\text{max}} \), and can be diagnosed as a drop in the detector voltage signal.

3.5. Summary

Each component of the new system: photodiodes detectors, detectors layout, electronics, and lenslet array re-imaging telescope, introduces a significant improvement to the conventional Shack-Hartmann wavefront sensor sensitivity, accuracy, size and sampling rate, compared to the previous generation of 8X8 sub-apertures high-bandwidth Shack-Hartmann wavefront sensor. Images of the assembled wavefront sensor case, and the electronic boards inside the case, are shown in Fig. 9. The assembled wavefront sensor case dimensions are 13x18x13", the case was designed to be robust and insensitive to vibrations and rough environments.

Fig. 9. Wavefront sensor case, containing the detector board, transimpedance amplifiers board, summation/subtraction board, and two dimensional linear translation stage, left. An image of the electronic boards inside the wavefront sensor case, right.

4. WAVEFRONT RECONSTRUCTION ALGORITHM

An essential part of a wavefront sensor system is the wavefront reconstruction algorithm, used to reconstruct and evaluate the instantaneous aberrated wavefront phase. As previously mentioned, the Shack-Hartmann wavefront sensor measures local gradients of the wavefront, \( S_x \) and \( S_y \), which are integrated to calculate the wavefront, \( \Phi \), and Eq. (4) can be re-written as

\[ \psi \Phi(x, y) = \tilde{S}(x, y) \]  \hspace{1cm} (14)

The incident wavefront is described in terms of the Cartesian coordinates \((x, y)\).

For a real-time AO system, the wavefront reconstruction algorithm is integrated in the conjugate constructor (CC) processor, controlling the closed-loop system. Hence it is really important to reconstruct the wavefront accurately and efficiently, and with minimum system latency.

Different solution methods have been developed by a number of authors. For our analysis, Fried geometry was adapted, where each PSD was represented as a square sub-aperture (cell) with the measured orthogonal phase gradients.
in its center, and the four estimated phases (nodes) lie in the sub-aperture corners. Denoting the elements by \((i, j)\) notation, each sub-aperture was represented as shown in Fig. 10, left. The grid array for the 10X10 sensor layout is shown in Fig. 10, right. In our case, the grid array has 68 cells and 89 nodes.

\[
\frac{\partial \Phi}{\partial x} = S_x, \quad \frac{\partial \Phi}{\partial y} = S_y, \quad \text{and} \quad \frac{\partial^2 \Phi}{\partial x \partial y} \rightarrow 0
\]

(15), (16), and (17)

Then, the least square estimation can be written as,

\[
\Delta = \left( \frac{\partial \Phi}{\partial x} - S_x \right)^2 + \left( \frac{\partial \Phi}{\partial y} - S_y \right)^2 + \left( \frac{\partial^2 \Phi}{\partial x \partial y} - 0 \right)^2 \rightarrow \text{Minimum}
\]

(18)

The third term is designed to suppress high spatial frequencies that can be present in each sub-aperture, and to make sure that Eq. (18) has a unique solution up to an arbitrary constant.

A second order central difference method can be used to discretize the partial derivative, and Eqs. (15)-(17) can be re-written using matrix notation as,

\[
A_x \Phi = S_x, \quad A_y \Phi = S_y, \quad \text{and} \quad A_{xy} \Phi = 0
\]

(19), (20), and (21)

where, \(A_x\), \(A_y\), and \(A_{xy}\) are matrix derivative operators \(\frac{\partial \Phi}{\partial x}\), \(\frac{\partial \Phi}{\partial y}\), and \(\frac{\partial^2 \Phi}{\partial x \partial y}\), respectively. The operators are rectangular sparse matrices with only 4 non-zero elements at each row. Each row or column of the matrix represents a different cell or a node, respectively. For the 10x10 wavefront sensor, the matrix dimensions are [68x89]. Using the matrix notation Eq. (18) can be re-written as

\[
\Delta = (A_x \Phi - S_x)^2 + (A_y \Phi - S_y)^2 + (A_{xy} - 0)^2 \rightarrow \text{Minimum}
\]

(22)

Taking the derivative with respect to \(\Phi\), and imposing either a zero wavefront value on one of the nodes, \(\Phi(x_0, y_0) = 0\), or requiring that \(\int_{Ap} \Phi(x, y) dxdy = 0\), so that a unique solution can be found, Eq. (18) can be finally written as,

\[
\Phi = C_x S_x + C_y S_y
\]

(23)

where the reconstruction matrices, \(C_x = A^{-1}A_x^T\) and \(C_y = A^{-1}A_y^T\), are independent of the input measured slopes, and are needed to be calculated once for a given sensor layout configuration and stored. If needed, the reconstruction matrices, \(C_x\) and \(C_y\) can be stored directly on a digital multiplication board integrated in a real time AO system.

5. TWO-DIMENSIONAL HEATED JET FACILITY

Experiments were conducted in the heated-jet facility at the University of Notre Dame to verify the accuracy of the newly designed wavefront sensor. A detailed description of the facility geometrical dimensions, flow measurements,
smoke visualization and phase-averaged temperature profile of the jet can be found in Ref. 18-20. Here, we provide a short description of the optical characteristics of the flow only.

The heated jet has core velocity, \( U_c \), of approximately 7 m/s. The jet's response to a 240 Hz forcing is to regularize the jet's most-unstable Kelvin-Helmholtz instability in the jet's two bound shear layers. The forcing also regularizes the first pairing, resulting in the formation of 120 Hz sub-harmonic, large-coherence-length flow structures. Without acoustic forcing, the jet has a random OPD peaks pattern as a function of time, as shown in Fig. 11, left. Acoustic forcing regularizes both the amplitude and temporal frequency of the OPD pattern, as shown in Fig. 11, right. This predictable, repeatable OPD pattern was used to inter-calibrate and compare the wavefront sensor measurements for this study.

Fig. 11. Time series of experimentally measured OPDs from propagation through a 2-D jet without acoustic forcing\(^{19}\), left.
With acoustic forcing\(^{20}\), right.

6. TESTS AND RESULTS

The high bandwidth wavefront sensor was tested in the University of Notre Dame forced heated jet facility, and compared to a Wavefront Sciences CLAS-2D wavefront sensor which has a 33X44 sub-apertures resolution and uses a conventional CCD array camera to detect spot displacement in the focal plane (cf. above). The test optical setup, sensor calibration, data processing and results are presented in Ref. 21. Here, only relevant results are presented.

Fig. 12. Typical instantaneous wavefronts realizations (2D-view) obtained with the high-bandwidth wavefront sensor for a wave propagating in X streamwise direction; \( 1.2 < X/D < 5.2; -1 < Y/D < 1; D = \frac{\lambda}{2} \).

Although the sensor is able to sample wavefronts at sampling speed of 104 kHz, for this test raw voltages from the 10x10 sensor were sampled at the lower sampling rate of 24 kHz; these signals were then post-processed to calculate local slopes, and then used to construct instantaneous wavefronts based on the algorithm presented in Section 4.

The convective nature of the regularized aberration in the streamwise direction, \( x \), and the presumption that the jet is essentially two-dimensional, is well demonstrated by the structure propagating through the beam aperture. Four typical instantaneous, two-dimensional wavefronts realizations are presented in Fig. 12. An initial aberration structure roll-up is
shown to start developing around $X/D = 1$, Fig. 12 (b), reaching a maximum aberration amplitude of approximately 0.2 $\mu$m around $X/D = 2$, Fig. 12 (c), and starts breaking down thereafter, Fig. 12 (d). Mean OPD$_{rms}$ and OPD$_{p-p}$ values were 0.059 $\mu$m and 0.25 $\mu$m, respectively.

![Wavefronts](image)

Fig. 13. Comparison of phase-locked averaged wavefront measurements: High-bandwidth sensor (solid line), CLAS-2D sensor (dash line); ($0.625 < X / D < 2.5$; $D = 1/2$).

In order to both validate and test the spatial resolution of the high bandwidth wavefront sensor, wavefronts from the 10x10 sensor were compared to those from the wavefront sciences CLAS-2D wavefront sensor, which although having a slower capturing rate, has a higher spatial resolution of 33x44 sub-apertures. Due to the periodic nature of the acoustically forced heated jet, a phase-locked averaging was performed to compare two sensors. Phase-lock averaging over 400 cycles was performed for 12 phase angles with separation between angles of 30°. In order to compare the two wavefront sensors, one-dimensional slices of the two-dimensional wavefronts were used. A comparison between the two sensors for the selected forcing phase angles is presented in Fig. 13. The comparison between the two wavefront sensors reveals that, although the high bandwidth wavefront sensor has only a modest spatial resolution of 10x10 sub-apertures, it captured and reconstructed all the essential large-scale features of the two-dimensional convecting aberrating structure. Further, despite the small peak-to-peak aberration amplitude, an average of OPD$_{p-p}$ = 0.11 $\mu$m for both sensors, the high bandwidth sensor was able to resolve the peak-to-peak aberration amplitude correctly when compared with the higher spatial resolution CLAS-2D wavefront sensor. Wavefront slices from the high bandwidth wavefront sensor were fit with a cubic polynomial to compare them with the wavefront slices acquired by the CLAS-2D wavefront sensor; for each forcing phase angle a spatial error was calculated based on the equation

$$\text{Error} = \sqrt{\frac{\sum_{i=1}^{N}(\text{OPD}_{10x10}^i - \text{OPD}_{\text{2D}}^i)^2}{N}}$$

(24)

where $N$ is the number of spatial points in the spanwise direction $x$. The calculated error for each phase is presented in Fig. 13. A mean error of 0.0176 $\mu$m was calculated for these 12 phase angles, this value is within 20% of the mean of OPD$_{p-p}$ both wavefront sensors.

7. CONCLUSION

An analog high-bandwidth, two-dimensional, wavefront sensor was developed and tested against a commercially available wavefront sensor. The analog nature of the new sensor enables high sampling rates to capture aero-optical convecting aberrations. Currently, the sensor's sampling rates are demonstrated to be greater than 100 kHz, which is essential for investigating aero-optical effects and developing adaptive-optics systems that can mitigate high temporal frequencies.
The spatial and temporal resolutions of the new sensor were tested on an acoustically forced heated jet facility and compared to the conventional wavefront sensor, which has a higher spatial resolution of 33x44 sub-aperture, compared to the 10x10 sub-apertures resolution of the high-bandwidth sensor. It was shown that despite the modest spatial resolution, the new sensor was capable of resolving the essential spatial character of aberrating structures.

The wavefront sensor can be used in two major ways. The first is as part of a conventional real (or pseudo-real) time adaptive optics system to diagnose and compensate for optical disturbances at high temporal rates, while maintaining the system stability. Secondly, the wavefront sensor can be used as a diagnostic tool, investigating optical disturbances associated with turbulent flows, up to a maximum frequency (at present) of 52 kHz limited by the Nyquist criterion; the current limit is due to the bandwidth limitation of presently used data acquisition system.

ACKNOWLEDGMENTS

These efforts were sponsored by the Office of Naval Research, ONR under Grant Number N00014-07-1-0291. The U.S. Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation thereon.

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